Why Can the Yield Curve Predict Output Growth, Inflation, and Interest Rates? An Analysis with an Affine Term Structure Model[☆]

Hibiki Ichiue[*]

Financial Markets Department, Bank of Japan

First Draft: July, 2003
Revised: July, 2007

Abstract

The literature provides evidence that term spreads help predict output growth, inflation, and interest rates. This paper explains these predictability results by using an affine term structure model with observable macroeconomic factors. The results suggest bond holders are willing to receive lower term premia during higher inflation regimes. This negative correlation causes term spreads to react to inflation shocks, which have persistent effects on the variables and prove useful for prediction. We also find that term spreads using the short end of the yield curve have less predictive power than many other spreads. This is attributed to monetary policy inertia.

JEL classification: E43; E52

Keywords: Term structure, Monetary policy, The expectations hypothesis, VAR

[☆] This paper is a revised version of Chapter 2 of my doctoral dissertation submitted to University of California, San Diego in August, 2005. I am especially grateful to my dissertation advisor, James D. Hamilton for his valuable support and comments. I would like to thank Marjorie Flavin, Akira Ieda, Bruce Lehmann, Nobuyuki Oda, Keiichi Tanaka, Alan Timmermann, David Vestin, Kenneth West, two anonymous referees and participants in the presentations at UCSD, European Central Bank, Yokohama National University and Bank of Japan. I am also grateful to Monica Piazzesi for answering my questions on her papers. The views expressed here are mine, and not necessarily of Bank of Japan.
[*] Address: 2-1-1 Nihonbashi-Hongokucho Chuo-ku Tokyo 103-8660 Japan, Tel.: +81-3-3279-1111 (Bank of Japan), Fax: +81-3-5203-7187, *E-mail address*: hibiki.ichiue@boj.or.jp

1. Introduction

Many studies in the literature provide evidence that interest rate term spreads contain information about three different future economic variables: output growth, inflation, and interest rates, for various sample periods and countries. However, the literatures examining the predictability of these three variables have been quite separate. Studies of the predictability of interest rates have been mainly conducted to test a popularly-held classic theory, namely the expectations hypothesis, which has been repeatedly rejected in the literature such as Campbell and Shiller (1991). The literature on the predictability of inflation rate also has a long history following Fama's (1975) classic study. Many researchers have also investigated the predictability of output growth after Stock and Watson (1989) found that the term spread plays an important role in their index of economic leading indicators.

Although there is an extensive literature providing evidence and explanations for each of these empirical predictabilities regarding output growth, inflation, and interest rates, no paper has yet tried to analyze the interaction among them. The main purpose of this paper is to integrate these predictability results in an attempt to answer an important question: why does the term structure predict future movements in economic variables?[1]

An affine term structure model (ATSM) with observable economic factors is used as our main tool. There have been a number of studies following Ang and Piazzesi's (2005) introduction of this type of model to investigate the relationship between macroeconomic variables and the term structure, for example, Hordahl, Tristani and

[1] Estrella (2005) uses a theoretical model to explain the empirical predictabilities regarding output growth and inflation rate. His model, however, seems too restricted to explain these facts. For example, the model assumes the expectations hypothesis, which has been repeatedly rejected in the literature. In fact, as will be seen, we find that the failure of the hypothesis is an important property of the term structure of interest rate for predicting future movements in economic variables.

Vestin (2006) and Wu (2006). Many of these studies depend on macroeconomic theories to restrict their models so that the results can be interpreted more easily. Furthermore, these models typically use latent variables other than observable variables, and interpret the latent factors as variables such as the monetary policy authority's inflation target.

Conversely, Ang, Piazzesi and Wei (2006) use only observable variables, and they do not use macroeconomic theory other than the no-arbitrage assumption to restrict their model. This type of model can be interpreted either as a VAR with no-arbitrage restrictions or as an ATSM with observable factors that follow a VAR process. In this paper, we call this type of model a VAR-ATSM for convenience. Ang, Piazzesi and Wei use their VAR-ATSM to examine the predictability of output growth using term spreads. We follow this basic idea, which we extend to include the predictabilities of inflation and short rates.[2] Although their basic idea is very useful, some of their assumptions are not suitable to our purpose here. Ang, Piazzesi and Wei try to identify good forecasting models by comparing the predictive powers, specifically the rolling out-of-sample forecasting performances, of various combinations of regressors. Their parsimonious VAR(1) model only with three factors may be appropriate for such an exercise. Our aim, however, is to shed light on the source of the predictability by analyzing the relationship between impulse response functions and R^2s. For this reason, we use a VAR with more lags and more variables, following the VAR literature. In fact, as will be seen, we find the added variable, the inflation rate, is the most crucial factor for the predictabilities.

The rest of this paper is organized as follows. Section 2 presents stylized facts

[2] Before Ang, Piazzesi and Wei (2006), several papers use term structure models with only latent factors for analyzing predictability using term spreads. For example, Hamilton and Kim (2002) use a two-factor term structure model to explain the predictability of output growth. Since these models use only latent factors, however, they have only limited value for analyzing the relationships between the term structure and macroeconomic variables.

from simple OLS results. Section 3 introduces the VAR-ATSM. Estimation methods and results are considered in Section 4. Section 5 uses impulse response functions and model-implied R^2s, which can be obtained from the estimated VAR-ATSM, to explain why term spreads predict well. Section 6 concludes.

2. Simple OLS Results

The empirical studies in the literature examine the predictive power of term spreads for future output growth, inflation, and interest rates using distinct types of regressions. In order to put the empirical results for predicting the different variables on a consistent basis, we use the regressions below,

$$g_{t+h} = \alpha + \beta(r_t^{(n)} - r_t^{(m)}) + \varepsilon_{t+h}; \tag{1}$$

$$\pi_{t+h} = \alpha + \beta(r_t^{(n)} - r_t^{(m)}) + \varepsilon_{t+h}; \tag{2}$$

$$r_{t+h}^{(1)} = \alpha + \beta(r_t^{(n)} - r_t^{(m)}) + \varepsilon_{t+h}; \tag{3}$$

for various combinations of h, n, and m (h = 1,2,…,12; n, m = 2, 4, 8, 12, 16, 20, and $n > m$), where g_t is the real GDP growth rate from $t-1$ to t, π_t is the inflation rate of GDP deflator from $t-1$ to t, and $r_t^{(n)}$ is the n-period nominal discount rate at t obtained from CRSP.[3] Following Ang, Piazzesi and Wei (2006), U.S. quarterly data from 1964:Q1 to 2001:Q4 are used. The variables g_t, π_t, and $r_t^{(n)}$ are all defined as rates per quarter. Regressands of (1) and (2) are continuously compounded marginal rates such as rates from $t+h-1$ to $t+h$. Although cumulative rates such as rates from t to $t+h$ are more popular in the literature, marginal rates are more convenient for

[3] CRSP (Center for Research in Security Prices, Graduate School of Business, the University of Chicago: www.crsp.uchicago.edu. All rights reserved.) Monthly US Treasury Database is used with permission.

specifying which part of the future the term spreads can predict well since cumulative rates are the average of marginal rates.

Panel (1-a) in Figure 1 displays the R^2s of OLS regressions (1)-(3) for selected term spreads as functions of h. The 20Q-1Q spread has predictive power for output growth, inflation, and short rates, at least for shorter horizons. This result is consistent with the literature, which argues that term spreads between 5-year (or 10-year) and 3-month rates predict well. But surprisingly we found that term spreads without the 1Q rate perform better than the 20Q-1Q spread in many cases. For example, (1-a) shows that the performance of the 12Q-8Q spread is superior, except for predicting output growth rates at shorter horizons. On the other hand, spreads between short rates, such as the 2Q-1Q spread, are almost useless. Together, these facts seem to imply that term spreads using the short end of the yield curve have less predictive power. This is surprising because the existing literature pays little attention to spreads that exclude the short end of the yield curve, and several studies including Ang, Piazzesi and Wei (2006) argue that the best predictive performance is achieved by maximal maturity difference. Another notable feature of the graphs is the hump-shape traced out by the R^2s of the output growth regressions. This suggests that it is difficult to predict the output growth rate at short horizons. Why do term spreads have this kind of predictive power? Since the OLS results do not answer this question, we need a more structured model.

3. Affine Term Structure Models with Observable Factors

This section considers two ATSMs, a simple one-factor model which assists intuition and a VAR-ATSM used for the complete analyses.

3.1. A Simple Example of One Factor Model

Following Cochrane (2001, p.358), we consider a simple consumption-based inter-temporal capital asset pricing model (C-CAPM) with constant relative risk aversion (CRRA) utility, consumption equal to output in equilibrium, constant price level, and stochastic discount factor

$$M_{t+1} = e^{-\delta}\left(\frac{Y_t}{Y_{t+1}}\right)^{\rho} = \exp(-\delta - \rho g_{t+1})\,. \tag{4}$$

Here δ is the subjective discount rate, ρ is the coefficient of relative risk aversion satisfying $\rho > 0$, and Y_t is the output at *t*. Suppose the output growth rate, the only factor, obeys an AR(1) process around zero as

$$g_{t+1} = \phi g_t + \sigma u_{t+1}, \tag{5}$$

where $0 \le \phi < 1$, $\sigma > 0$, and $u_{t+1} \sim N(0,1)$ i.i.d. Then the stochastic discount factor (4) follows a conditional log-normal distribution as

$$M_{t+1} = \exp(-\delta - \rho\phi g_t - \rho\sigma u_{t+1})\,. \tag{6}$$

The coefficient corresponding to the shock $\rho\sigma$ is called the market price of risk. Since the market price of risk is time-invariant and positive, a positive output growth shock has a negative effect on M_{t+1}. This is consistent with a role for bonds as a consumption (output) hedge. That is, when the future output growth rate is higher, consumers feel that future cash flows are less important.

Bond prices satisfy

$$Q_t^{(n)} = E_t[M_{t+1} Q_{t+1}^{(n-1)}], \tag{7}$$

for $n = 1, 2, \ldots$, where $Q_t^{(n)}$ is the n-period bond price with $Q_t^{(0)} = 1$ and $Q_t^{(1)} = \exp(-r_t^{(1)})$. When $n = 1$, we have the Euler equation

$$\exp(-r_t^{(1)}) = Q_t^{(1)} = E_t[M_{t+1}]. \tag{8}$$

By substituting (6) into (8), the short rate is proved to be represented as a linear function of the output growth as

$$r_t^{(1)} = \overline{r} + \rho\phi g_t \tag{9}$$

where $\overline{r} = \delta - (\rho\sigma)^2 / 2$. By using (9), the stochastic discount factor (6) can be rewritten as

$$M_{t+1} = \exp\{-r_t^{(1)} - (\rho\sigma)^2 / 2 - \rho\sigma u_{t+1}\}. \tag{10}$$

Note that since the short rate obeys an AR(1) process and the market price of risk is time-invariant, this model is mathematically identical to that of Vasicek (1977).

By using the fundamental asset pricing equation (7), closed forms for the discount rates $r_t^{(n)}$ can be derived as affine functions of the factor g_t:

$$r_t^{(n)} = a^{(n)} + b^{(n)} g_t, \tag{11}$$

for $n = 1, 2, \ldots$, where $a^{(n)}$ is a constant and the factor loading $b^{(n)}$ satisfies

$$b^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1}\phi^j b^{(1)} = \frac{1}{n}\frac{1-\phi^n}{1-\phi}\phi\rho, \tag{12}$$

which can be interpreted as the sensitivity of longer rates $r_t^{(n)}$ to the factor. The restriction $0 \le \phi < 1$ guarantees that the factor loading $b^{(n)}$ does not diverge with the maturity n. This implies that the movement of short rates is more volatile than that of long rates as observed in U.S. data.

By using (12), the term premium can be obtained as

$$r_t^{(n)} - \frac{1}{n}\sum_{j=0}^{n-1} E_t[r_{t+j}^{(1)}] = a^{(n)} - a^{(1)}. \tag{13}$$

The term premium is therefore time-invariant, i.e. the expectation hypothesis holds. In this case, movements in long rates $r_t^{(n)}$ depend only on movements in average expected short rates $n^{-1}\sum_{j=0}^{n-1} E_t[r_{t+j}^{(1)}]$.

An important caveat of this simple model is time-invariant market price of risk and term premium, which are often rejected in the literature. Since the term premium affects the relationship between short and long rates, i.e. movements in term spreads, this may be very important for examining the predictive power of term spreads. In the next subsection, we will introduce a more general and less restricted model.

3.2. The VAR-ATSM

In this subsection, the simple AR(1) process (5) of output growth rate is generalized to a VAR(4) process with four factors, and the market price of risk is allowed to be time-varying. The VAR-ATSM can be interpreted as either a VAR model with no-arbitrage restrictions or an ATSM with observable factors that follow a VAR process. Let's start by considering the factor VAR.

Four variables are used as factors: the output growth rate g_t, the inflation rate π_t, the short rate $r_t^{(1)}$, and a benchmark term spread s_t. For s_t, we use the term spread between ten-year Treasury bond YTM at the end of quarter t and $r_t^{(1)}$. These four macroeconomic variables are assumed to follow a VAR(4) process,

$$\mathbf{x}_t = \mathbf{c} + \mathbf{\Phi}_1\mathbf{x}_{t-1} + \mathbf{\Phi}_2\mathbf{x}_{t-2} + \mathbf{\Phi}_3\mathbf{x}_{t-3} + \mathbf{\Phi}_4\mathbf{x}_{t-4} + \boldsymbol{\varepsilon}_t, \tag{14}$$

where $\mathbf{x}_t = (g_t, \pi_t, r_t^{(1)}, s_t)'$ and $\boldsymbol{\varepsilon}_t = (\varepsilon_{g,t}, \varepsilon_{\pi,t}, \varepsilon_{r,t}, \varepsilon_{s,t})'$. Following the VAR literature, we interpret $r_t^{(1)}$ as a proxy for the monetary policy instrument. Ang, Piazzesi and Wei (2006) use a simpler model than ours. They use a VAR with only one lag and three variable which do not include the inflation rate.

To give a structural interpretation to the VAR, we use a recursive structure with the variables ordered as $(g_t, \pi_t, r_t^{(1)}, s_t)$. That is, $\boldsymbol{\varepsilon}_t = \boldsymbol{\Sigma}\mathbf{u}_t$ where exogenous shocks $\mathbf{u}_t = (u_{g,t}, u_{\pi,t}, u_{r,t}, u_{s,t})'$ are $N(\mathbf{0},\mathbf{I})$ i.i.d., and $\boldsymbol{\Sigma}$ is lower-triangular with positive diagonal elements. We call the elements of $\mathbf{u}_t$ output growth, inflation, monetary policy, and spread shocks, respectively. Since significant responses of g_t and π_t to contemporaneous interest rates are implausible, our ordering places them before $r_t^{(1)}$ and s_t. The order of g_t and π_t should not seriously affect the empirical results, since the correlation between $\varepsilon_{g,t}$ and $\varepsilon_{\pi,t}$ is small as shown later. We follow much of the literature in assuming that the short rate (the monetary policy authority) does not respond to the term spread (bond market) contemporaneously.[4] As will be seen in Section 5, the impulse responses seem to be reasonable, and support our recursive assumption.

The VAR in (14) can be rewritten in companion form,

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{x}_{t-2} \\ \mathbf{x}_{t-3} \end{bmatrix} = \begin{bmatrix} \mathbf{c} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_2 & \boldsymbol{\Phi}_3 & \boldsymbol{\Phi}_4 \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{t-1} \\ \mathbf{x}_{t-2} \\ \mathbf{x}_{t-3} \\ \mathbf{x}_{t-4} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Sigma} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}_t \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}.$$

or

[4] Leeper, Sims, and Zha (1996) discuss this issue in detail.

$$\mathbf{X}_t = \tilde{\mathbf{c}} + \tilde{\mathbf{\Phi}}\mathbf{X}_{t-1} + \tilde{\mathbf{\Sigma}}\tilde{\mathbf{u}}_t, \tag{15}$$

where $\mathbf{X}_t = (g_t, \pi_t, r_t^{(1)}, s_t, \ldots, g_{t-3}, \pi_{t-3}, r_{t-3}^{(1)}, s_{t-3})'$ is the 16×1 state vector.

The stochastic discount factor is defined as

$$M_{t+1} = \exp\left(-r_t^{(1)} - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t'\mathbf{u}_{t+1}\right) \tag{16}$$

where $\boldsymbol{\lambda}_t = (\lambda_{g,t}, \lambda_{\pi,t}, \lambda_{r,t}, \lambda_{s,t})'$ are the market prices of risk. The vector $\boldsymbol{\lambda}_t$ is an affine function of the current economic variables $\mathbf{x}_t = (g_t, \pi_t, r_t^{(1)}, s_t)'$ as $\boldsymbol{\lambda}_t = \boldsymbol{\gamma} + \boldsymbol{\delta}\mathbf{x}_t$ for a 4×1 vector $\boldsymbol{\gamma}$ and a 4×4 matrix $\boldsymbol{\delta}$.

By using the fundamental asset pricing equation (7), we can obtain closed forms for discount rates:

$$r_t^{(n)} = a^{(n)} + \mathbf{b}^{(n)}{}'\mathbf{X}_t, \tag{17}$$

for n = 1, 2, … where $a^{(n)}$ is a constant,

$$\mathbf{b}^{(n)}{}' = \frac{1}{n}\mathbf{e}_3{}'\sum_{j=0}^{n-1}(\tilde{\mathbf{\Phi}} - \tilde{\mathbf{\Sigma}}\tilde{\boldsymbol{\delta}})^j \quad , \tag{18}$$

$$\tilde{\boldsymbol{\gamma}} = \begin{bmatrix} \boldsymbol{\gamma} \\ \mathbf{0} \end{bmatrix}, \quad \tilde{\boldsymbol{\delta}} = \begin{bmatrix} \boldsymbol{\delta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{19}$$

and $\mathbf{e}_j$ is the j th column of the 16×16 identity matrix. The term premium can be obtained as

$$r_t^{(n)} - \frac{1}{n}\sum_{j=0}^{n-1}E_t[r_{t+j}^{(1)}] = a^{(n)} - a^{(1)} + \frac{1}{n}\mathbf{e}_3{}'\sum_{j=0}^{n-1}[(\tilde{\mathbf{\Phi}} - \tilde{\mathbf{\Sigma}}\tilde{\boldsymbol{\delta}})^j - \tilde{\mathbf{\Phi}}^j]\mathbf{X}_t. \tag{20}$$

Thus the term premium depends on $\mathbf{X}_t$ and is time-varying in general. But as an important special case, when $\boldsymbol{\delta} = \mathbf{0}$, the premium is time-invariant.

4. Estimation

The VAR-ATSM has 98 parameters consisting of 78 from the VAR ($\mathbf{c}$, $\mathbf{\Phi} \equiv [\mathbf{\Phi}_1\, \mathbf{\Phi}_2\, \mathbf{\Phi}_3\, \mathbf{\Phi}_4]$, and $\mathbf{\Sigma}$) and 20 in market prices of risk ($\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$). GMM is used to estimate all parameters simultaneously. Moment conditions are constructed by assuming that the three types of error are orthogonal to their instruments. The first of these are the VAR errors $\boldsymbol{\varepsilon}_t$ where the instruments are a constant, $\mathbf{x}_{t-1}$, $\mathbf{x}_{t-2}$, $\mathbf{x}_{t-3}$, and $\mathbf{x}_{t-4}$. The second type is the error of the covariance matrix of the VAR, $\text{vech}(\mathbf{\Sigma\Sigma}' - \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')$, the sample mean of which is forced to equal to zero. Note that the moment conditions corresponding to these two types of errors are exactly the same as in OLS. The third type consists of the discount rate pricing errors $\mathbf{v}_t = [\nu_t^{(2)}\ \nu_t^{(4)}\ \nu_t^{(8)}\ \nu_t^{(12)}\ \nu_t^{(16)}\ \nu_t^{(20)}]'$ where $\nu_t = r_t^{(n)} - (a^{(n)} + \mathbf{b}^{(n)}{}'\mathbf{X}_t)$. A constant, $\mathbf{x}_{t-1}$, and $\mathbf{x}_{t-2}$ are used as instruments for this type of moment. That is, we assume that the pricing errors are unforecastable by the old information such as $\mathbf{x}_{t-1}$ or $\mathbf{x}_{t-2}$. Now we have 132 moment conditions, which are sufficient for identifying 98 parameters. We use the sample period 1964:1Q-2001:4Q, the same as was used for the OLS regressions in Section 2.

The parameter space is restricted by forcing the eigenvalues of $\tilde{\mathbf{\Phi}} - \tilde{\mathbf{\Sigma}}\tilde{\boldsymbol{\delta}}$ to be less than or equal to unity in modulus.[7] From (18), the factor loading $\mathbf{b}^{(n)}$ can be considered as the average of $\mathbf{e}_3{}'(\tilde{\mathbf{\Phi}} - \tilde{\mathbf{\Sigma}}\tilde{\boldsymbol{\delta}}\ )^j$; j = 0, 1,…, n-1. So this restriction

[7] When a restriction binds, the spectral density matrix at frequency zero is not guaranteed to be the optimal weighting matrix in GMM. To solve this problem, we use the binding restriction to substitute out a parameter in advance. Inference will then be correct when we use the obtained non-restricted GMM to estimate parameters. The estimate and standard error of the substituted parameter are obtained by substituting out another parameter and re-estimating.

guarantees that the factor loading does not diverge as maturity n increases. Note that this restriction is the generalization of $0 \le \phi < 1$ in the simple one-factor model as discussed after (12).

The estimate of $\boldsymbol{\Sigma}$ is reported in Table 1. The diagonal elements of $\boldsymbol{\Sigma}$ are much higher than the others in general, which implies that correlations among the reduced VAR errors are small, but the contemporaneous effect of the short rate shock $u_{r,t}$ on the term spread s_t is too large to be ignored. The first diagonal element corresponding to the output growth is the largest one, and this is about three times as large as the second largest one, that for the inflation shock.

Table 2 reports the estimates for $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$. Seven out of 16 estimates of $\boldsymbol{\delta}$ are significantly different from zero at size of 5%. Among these significant parameters, the (1,1) and (1,2) elements of $\boldsymbol{\delta}$, δ_{11} and δ_{12} have the most influence on the term structure. The reason for this is considered using equation (18) as follows, since it is difficult to interpret $\boldsymbol{\delta}$ directly. How the factors $\mathbf{X}_t$ affects the term structure depends only on the factor loadings $\mathbf{b}^{(n)}$, which depend on $\tilde{\boldsymbol{\Phi}} - \tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{\delta}}$ according to (18). So the influence of $\boldsymbol{\delta}$ on the term structure depends on $\tilde{\boldsymbol{\Sigma}}$ (i.e. $\boldsymbol{\Sigma}$). As can be seen in Table 1, the (1,1) element of $\boldsymbol{\Sigma}$, which can be interpreted as the volatility of output growth shock, is much larger than the others. So the first row of $\boldsymbol{\delta}$ is the most influential. Among the estimates in the first row, only δ_{11} and δ_{12} are significantly different from zero. In fact, as will be discussed in the next section, δ_{12} plays a key role in the predictive power of term spreads, while δ_{11} does not. The positive sign of δ_{12} implies that, when the inflation rate π_t is higher, $\lambda_{g,t}$ is higher and bond holders are willing to pay a higher premium for an output hedge, which results in a lower term premium.

5. Impulse Response Functions and the Predictive Power of Term Spreads

From the VAR-ATSM estimated in the previous section, we can calculate the optimal forecasts conditional on the 16 state variables in $\mathbf{X}_t$. However, our main interest lies not in forecasts conditional on this large number of variables, but in forecasts conditional on the term spread alone, in line with the regressions in (1)-(3). For our purpose, in subsection 5.1, we first derive the model-implied R^2s, which are functions of parameters in the VAR-ATSM. They will be used to shed light on the source of the predictive power of term spreads in subsection 5.2.

5.1. Model-implied R^2s

Since the factors $\mathbf{x}_t = (g_t, \pi_t, r_t^{(1)}, s_t)'$ follow the VAR process specified in (14) and discount rates $r_t^{(n)} = a^{(n)} + \mathbf{b}^{(n)}{}'\mathbf{X}_t$ and term spreads $r_t^{(n)} - r_t^{(m)}$ are affine functions of $\mathbf{X}_t$, the corresponding impulse response functions can be calculated as functions of $\mathbf{\Phi}$, $\mathbf{\Sigma}$, and $\boldsymbol{\delta}$. Using these impulse response functions, the factors and the term spread can be represented in VMA(∞) form with identified exogenous shocks. For example, the output growth g_t and the term spread $r_t^{(n)} - r_t^{(m)}$ can be represented as

$$g_t = \bar{g} + \sum_{j=0}^{\infty} \psi_{gg,j} u_{g,t-j} + \sum_{j=0}^{\infty} \psi_{g\pi,j} u_{\pi,t-j} + \sum_{j=0}^{\infty} \psi_{gr,j} u_{r,t-j} + \sum_{j=0}^{\infty} \psi_{gs,j} u_{s,t-j}\,, \qquad (21)$$

$$r_t^{(n)} - r_t^{(m)} = \overline{r^{(n)} - r^{(m)}} + \sum_{j=0}^{\infty} \kappa_{g,j}^{(n,m)} u_{g,t-j} + \sum_{j=0}^{\infty} \kappa_{\pi,j}^{(n,m)} u_{\pi,t-j} + \sum_{j=0}^{\infty} \kappa_{r,j}^{(n,m)} u_{r,t-j} + \sum_{j=0}^{\infty} \kappa_{s,j}^{(n,m)} u_{s,t-j}$$

(22)

where $\bar{g}$ and $\overline{r^{(n)} - r^{(m)}}$ are the unconditional means of g_t and $r_t^{(n)} - r_t^{(m)}$, and $\boldsymbol{\psi}_{g,j} = (\psi_{gg,j}, \psi_{g\pi,j}, \psi_{gr,j}, \psi_{gs,j},)'$ and $\boldsymbol{\kappa}_j^{(n,m)} = (\kappa_{g,j}^{(n,m)}, \kappa_{\pi,j}^{(n,m)}, \kappa_{r,j}^{(n,m)}, \kappa_{s,j}^{(n,m)})'$ are impulse response functions, at *j* periods after the shocks, of the output growth rate g_t and the term spread $r_t^{(n)} - r_t^{(m)}$ respectively.

Future output growth g_{t+h} can be represented as

$$g_{t+h} = \hat{g}_{t+h|t} + \sum_{j=0}^{h-1} \psi_{gg,j} u_{g,t+h-j} + \sum_{j=0}^{h-1} \psi_{g\pi,j} u_{\pi,t+h-j} + \sum_{j=0}^{h-1} \psi_{gr,j} u_{r,t+h-j} + \sum_{j=0}^{h-1} \psi_{gs,j} u_{s,t+h-j} \quad (23)$$

where

$$\hat{g}_{t+h|t} = \bar{g} + \sum_{j=h}^{\infty} \psi_{gg,j} u_{g,t+h-j} + \sum_{j=h}^{\infty} \psi_{g\pi,j} u_{\pi,t+h-j} + \sum_{j=h}^{\infty} \psi_{gr,j} u_{r,t+h-j} + \sum_{j=h}^{\infty} \psi_{gs,j} u_{s,t+h-j} \quad (24)$$

is the optimal forecast of g_{t+h} conditional on $\mathbf{X}_t$.

Since $\mathbf{u}_t \sim N(\mathbf{0}, \mathbf{I})$ i.i.d., we can calculate the variances of the VAR variables, optimal forecasts for them, and term spreads. For instance, from (21), (22) and (24), we can obtain

$$\sigma_g{}^2 \equiv \mathrm{var}(g_t) = \sum_{j=0}^{\infty} \psi_{gg,j}{}^2 + \sum_{j=0}^{\infty} \psi_{g\pi,j}{}^2 + \sum_{j=0}^{\infty} \psi_{gr,j}{}^2 + \sum_{j=0}^{\infty} \psi_{gs,j}{}^2 , \quad (25)$$

$$(\sigma^{(n,m)})^2 \equiv \mathrm{var}(r_t^{(n)} - r_t^{(m)})$$
$$= \sum_{j=0}^{\infty} \kappa_{g,j}^{(n,m)2} + \sum_{j=0}^{\infty} \kappa_{\pi,j}^{(n,m)2} + \sum_{j=0}^{\infty} \kappa_{r,j}^{(n,m)2} + \sum_{j=0}^{\infty} \kappa_{s,j}^{(n,m)2} , \quad (26)$$

$$\sigma_{\hat{g},h}{}^2 \equiv \operatorname{var}(\hat{g}_{t+h|t}) = \sum_{j=h}^{\infty} \psi_{gg,j}{}^2 + \sum_{j=h}^{\infty} \psi_{g\pi,j}{}^2 + \sum_{j=h}^{\infty} \psi_{gr,j}{}^2 + \sum_{j=h}^{\infty} \psi_{gs,j}{}^2 \qquad (27)$$

The correlations between the variables can also be calculated by using the impulse response functions. For example, the correlation between future output growth rate g_{t+h} and the current term spread $r_t^{(n)} - r_t^{(m)}$ is represented as

$$\begin{aligned} &\operatorname{corr}(g_{t+h}, r_t^{(n)} - r_t^{(m)}) \\ &= \sum_{j=0}^{\infty} \frac{\psi_{gg,j+h} \kappa_{g,j}^{(n,m)}}{\sigma_g \sigma^{(n,m)}} + \sum_{j=0}^{\infty} \frac{\psi_{g\pi,j+h} \kappa_{\pi,j}^{(n,m)}}{\sigma_g \sigma^{(n,m)}} + \sum_{j=0}^{\infty} \frac{\psi_{gr,j+h} \kappa_{r,j}^{(n,m)}}{\sigma_g \sigma^{(n,m)}} + \sum_{j=0}^{\infty} \frac{\psi_{gs,j+h} \kappa_{s,j}^{(n,m)}}{\sigma_g \sigma^{(n,m)}} \end{aligned} \qquad (28)$$

Since R^2s of a simple linear regression equals a square of the correlation between the regressand and the regressor, the R^2 of regression (1) can be obtained by squaring (28) as

$$R_{g,h}^{2(n,m)} = \operatorname{corr}(g_{t+h|t}, r_t^{(n)} - r_t^{(m)})^2 = \operatorname{corr}(\hat{g}_{t+h|t}, r_t^{(n)} - r_t^{(m)})^2 . \qquad (29)$$

The second equation of (29) holds because the forecasting error of the optimal forecast $g_{t+h} - \hat{g}_{t+h|t}$ is unable to be predicted by any variable known at time *t*, such as $r_t^{(n)} - r_t^{(m)}$.

We can calculate the R^2s of the regressions (1)-(3) from the estimates of the parameters, since they are functions of parameters in our VAR-ATSM. We call these the model-implied R^2s. Equation (29) implies that if $r_t^{(n)} - r_t^{(m)}$ is a good predictor for future output growth, $r_t^{(n)} - r_t^{(m)}$ should respond to exogenous shocks in a similar way to $\hat{g}_{t+h|t}$. We investigate this by looking at the variance decomposition of $\hat{g}_{t+h|t}$ in the next subsection. Finally, as we can see from (28) and (29), the R^2s depend on the sum of products of the impulse response functions for regressands and regressors. Note that, in (28), indexes for ψ's start from *t+h*, not *t*, because future shocks $\mathbf{u}_{t+1},\ldots,\mathbf{u}_{t+h}$ are unpredictable. This implies that since the ψ's typically decay with the horizon *j*,

$r_t^{(n)} - r_t^{(m)}$ is a good predictor if it is responsive to recent shocks, i.e. κ's are large for smaller *j*.

5.2. Why do term spreads have predictive power?

Panel (1-b) in Figure 1 displays the model-implied R^2s from regressions (1)-(3) for three selected term spreads, and is the model-calculated analog of Panel (1-a). The results show that the model-implied R^2s replicate three properties of the sample R^2s. First, the 12Q-8Q spread performs better than the 20Q-1Q spread, except for output growth predictions at shorter horizons. Second, the 2Q-1Q spread is almost useless. Finally, it is difficult to predict output growth at 1Q ahead. It is therefore reasonable to try to explain the sample R^2s in terms of the factors that determine the model-implied R^2s.

Figure 2 shows the impulse response functions of the VAR variables g_t, π_t, $r_t^{(1)}$, and s_t to one unit exogenous shocks. In general, these results are consistent with those in the VAR literature. For example, (2-a) and (2-b) show that the short rate, the instrument of the monetary policy authority, responds positively to output growth and inflation shocks. Panel (2-c) demonstrates that the estimated monetary policy shock sharply reduces output growth. This shock also suppresses inflation rates in the long run. These results suggest that estimates of the monetary policy shock are consistent with the literature and reasonable. Further support is provided by Panel (2-d). As we discussed in Section 3, the most questionable part of our identification strategy may come from the contamination between the monetary policy shock and the spread shock. Panel (2-d) indicates that the estimated spread shock raises output growth and suppresses inflation. Since output growth and inflation should respond to a monetary policy shock in the same direction, the results in (2-d) suggest that the spread shock is not measuring a change in monetary policy.

Figure 3 shows variance decompositions of the optimal forecasts, where the variances of forecasts such as (27) are normalized to unity. As discussed in the previous subsection, this indicates which exogenous shocks should be useful for prediction. Panel (3-a) shows that the output growth shock dominates predictions of output growth at one quarter ahead. Then around 2-4 quarters ahead, the monetary policy shock is the most important. The importance of the inflation shock increases with the forecasting horizon, and this shock finally becomes most influential at 12 quarters ahead. These results are consistent with the impulse response functions in Figure 2. The output growth shock causes a sharp jump in output growth, but only in the short run. The monetary policy shock has a negative effect on output growth, but with 2-4 quarter lags. In the long run, the rise in the short rate induced by the inflation shock is persistent, and this acts to suppress output growth. Panels (3-b) and (3-c) show that the inflation shock is most important for predicting inflation and short rates at most horizons. Accordingly, the response of the term spread to the inflation shock is crucial for specifying the source of its predictive power, especially at longer horizons. Note that, as Figure 2 implies, the effects of exogenous shocks decay with the horizon. So we can also say that good predictors should respond to recent shocks rather than old shocks.

Figure 4 shows impulse response functions of selected discount rates and term spreads. There are three notable features. First, the effect of the inflation shock on the levels of the discount rates is highly persistent. In fact, the discount rates do not return to zero even after 40 quarters. Since good predictors should respond to recent shocks rather than old shocks, this is an important reason why levels of yield curves do not have great predictive power.

Second, discount rates with different maturities display different responses to recent shocks, while they respond to old shocks in similar ways. This implies that most movements in term spreads are due to recent shocks, because old shocks result in almost parallel shifts of the yield curve. In fact, Figure 4 illustrates the considerable dependence of both the 20Q-1Q and 12Q-8Q spreads on recent shocks. This is one reason why term spreads have predictive power.

Why do discount rates respond like this? We find that the time-varying market price of risk plays the following important role. As discussed in Section 4, it is the parameters corresponding to the effects of the output growth and inflation rates on the market price of output growth risk, δ_{11} and δ_{12}, that have the most influence on movements in term premia. Of these, only δ_{12} has a supportive role to play in the predictive relationship. As discussed, the inflation shock is the crucial element in the predictability, and a positive δ_{12} causes the market price of output growth risk to respond positively to the shock. In contrast to this, a positive δ_{11} reduces predictive power. As shown in (2-b), a positive inflation shock causes a decrease in the output growth rate, which has a negative effect on the market price of output growth risk. Since the effect from δ_{12} dominates the effect from δ_{11}, the market price of output growth risk responds positively and so the term premium responses negatively to the inflation shock.

In evaluating the effect from δ_{12}, the expectations hypothesis should be reconsidered. The expectations hypothesis states that the long rate is the average of expected short rates plus a time-invariant term premium. In (4-b), the short rate continues to rise up to around 20 quarters after the inflation shock. So, according to the hypothesis, the initial responses of long rates with maturities up to 20 quarters should be stronger than the response of the short rate. Since δ_{12} is positive, however, the inflation shock raises the market price of output growth risk, and so reduces the term premium. This is why long rates respond less strongly than the short rate in (4-b). This has a significant effect on the predictive power of term spreads. Panel (1-c) in Figure 1 displays model-implied R^2s for the case when $\delta_{12} = 0$. Surprisingly, the R^2s sharply decreased and almost disappear for the inflation rate and the short rate. This enables us to conclude that a positive δ_{12}, which can be interpreted in terms of bond holders' willingness to receive a lower premium for an output growth risk hedge during a higher inflation regime, is a key explanation for the predictive power of the term spread.

The last notable feature of Figure 4 is the lagged response of the 1Q rate (the monetary policy authority) to output growth and inflation shocks. Panel (4-a) shows that the immediate response of the 1Q rate to an output growth shock is the smallest among the discount rates, although the response of the 1Q rate is the largest several quarters ahead. Panel (4-b) shows that the immediate response of the 1Q rate to an inflation shock is smaller than that of the 2Q rate, and almost coincides with the response of the 8Q rate. These results are consistent with inertial behavior by the monetary policy authority, as empirically shown by, among others, Clarida, Gali, and Gertler (2000). Panel (4-a) and (4-b) also show the impulse response functions of 20Q-1Q and 12Q-8Q spreads to output growth and inflation shocks. The immediate response of the 20Q-1Q spread is much weaker than that of the 12Q-8Q spread because of the slow response of the 1Q rate. Since recent shocks are very important for predictive purposes, this can be concluded to be the reason behind the inferior performance of the 20Q-1Q spread compared to the 12Q-8Q spread.

Further support for this view is provided by the correlations between future predicted variables and current term spreads. Since model-implied R^2s are squares of these model-implied correlations, the correlations can be used to analyze why we found the R^2s shown in Figures 1. Equation (28) has four summed terms, each of which can be interpreted as the contribution of the corresponding exogenous shock to the predictive power of term spread. Figure 5 shows the contributions of exogenous shocks to the absolute values of the correlations with the 20Q-1Q and 12Q-8Q spreads. The output growth and inflation shocks contribute to the correlations with the 12Q-8Q spread rather than with the 20Q-1Q spread. These differences explain why the 12Q-8Q spread is useful for prediction. This result is consistent with the discussion in the previous paragraph.

Another notable feature of Figure 5 is the hump-shaped contribution of monetary policy shocks to output growth predictions. So it can be concluded that the hump-shape of the R^2s for the output growth predictions is attributable to the monetary policy shock. That is, the monetary policy shock affects output growth with a lag, while the term structure responds to the shock immediately. This difference in timing makes it harder for term spreads to help forecast output growth at short horizons.

6. Conclusion

We have three main findings. First, the time-varying market price of output growth risk, which is sensitive to the inflation rate, plays a key role in explaining the predictive power of term spread. When the inflation rate is higher, bond holders are willing to receive a lower premium for an output growth risk hedge. This causes term spreads to react to recent inflation shocks, which also proves useful for forming longer-run forecasts. Second, term spreads using the short end of yield curve have less predictive power than many spreads between longer rates. This fact is attributable to the inertial character of monetary policy. Finally, it is hard to predict output growth with term spreads at short horizons, because monetary policy shocks affect output growth with a lag while the term structure responds to the shock immediately.

References

Ang, A. and M. Piazzesi, 2005. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. Journal of Monetary Economics 50, 745-787.

Ang, A., M. Piazzesi and M. Wei, 2006. What does the yield curve tell us about GDP growth? Journal of Econometrics 131, 359-403.

Campbell, J. Y., and R. J. Shiller, 1991. Yield spread and interest rate movements: a bird's eye view. Review of Economic Studies 58, 495-514.

Clarida, R., J. Gali and M. Gertler, 2000. Monetary policy rules and macroeconomic stability: evidence and some theory. The Quarterly Journal of Economics 115, 147-180.

Cochrane, J. H., 2001. Asset pricing (Princeton University Press).

Estrella, A. 2005. Why Does the Yield Curve Predict Output and Inflation? The Economic Journal 115, 722-744.

Fama, E. F., 1975. Short term interest rates as predictors of inflation. American Economic Review 65, 269-282.

Hamilton, J. D., 1994. Time series analysis (Princeton University Press).

Hamilton, J. D. and D. H. Kim, 2002. A re-examination of the predictability of the yield spread for real economic activity. Journal of Money, Credit, and Banking 34, 2, 340-360.

Hordahl, P., O. Tristani, and D. Vestin, 2006. A joint econometric model of macroeconomic and term structure dynamics. Journal of Econometrics 131, 405-444.

Leeper, E. M., C. A. Sims, and T. Zha, 1996. What does monetary policy do? Brookings Papers on Economic Activity, 2, 1-63.

Stock, J. H., and M. W. Watson, 1989. New index of coincident and leading indicators. in O. Blanchard and S. Fischer, eds., NBER Macroeconomic Annual, Vol. 4 (MIT Press).

Vasicek, O., 1977. An equilibrium characterization of the term structure. Journal of Financial Economics 5, 177-188.

Wu, T., 2006. "Macro Factors and the Affine Term Structure of Interest Rates," working paper. Journal of Money, Credit, and Banking 38, 7, 1847-1876.

Table 1: Estimates of $\boldsymbol{\Sigma}$

	Shocks			
	$u_{g,t}$	$u_{\pi,t}$	$u_{r,t}$	$u_{s,t}$
g_t	0.0076	0	0	0
π_t	-0.0001	0.0025	0	0
$r_t^{(1)}$	0.0007	0.0004	0.0023	0
s_t	-0.0002	-0.0002	-0.0014	0.0012

$\boldsymbol{\Sigma}$ is estimated by GMM, as introduced in Section 4.

Table 2: Estimates of $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$

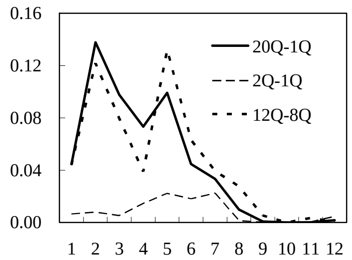
	$\boldsymbol{\gamma}$	$\boldsymbol{\delta}$			
		g_t	π_t	$r_t^{(1)}$	s_t
$\lambda_{g,t}$	-0.50 (0.43)	140* (22)	78* (32)	-26 (26)	-43 (63)
$\lambda_{\pi,t}$	-0.89 (0.90)	-99* (48)	62 (57)	-60 (51)	-177 (114)
$\lambda_{r,t}$	0.25 (0.25)	-23* (10)	-13 (11)	-30* (12)	-45* (17)
$\lambda_{s,t}$	0.67* (0.31)	-46 (29)	28* (14)	-30 (16)	-114* (26)
mean of factor		0.0080	0.0102	0.0159	0.0026
s.d. of factor		0.0089	0.0061	0.0065	0.0032

$\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are estimated by GMM, as introduced in Section 4. The estimates with * are significantly different from zero at 5%. Standard errors are in parentheses. Last two rows report means and standard deviations of the four factors.

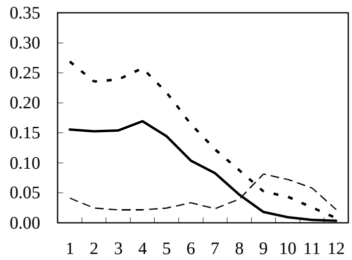
Figure 1: Sample and Model-implied R^2s

(1-a) Sample R^2s

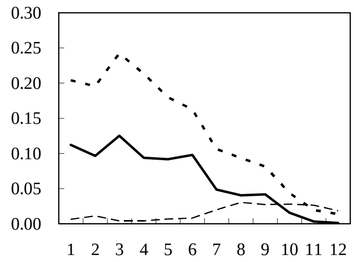
(i) Output growth rate



(ii) Inflation rate

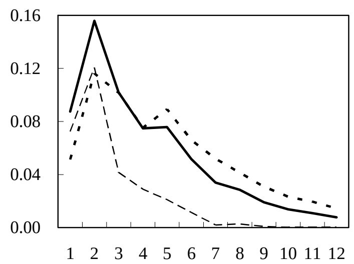


(iii) Short rate

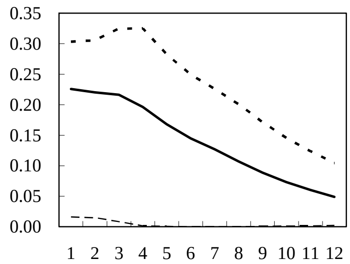


(1-b) Model-implied R^2s

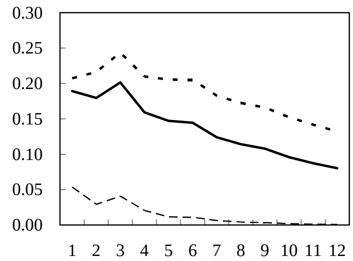
(i) Output growth rate



(ii) Inflation rate

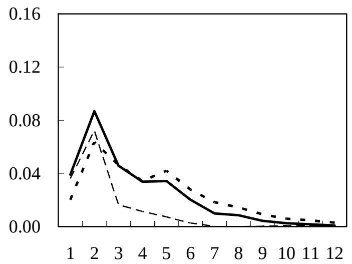


(iii) Short rate

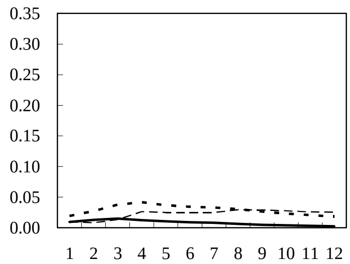


(1-c) Model-implied R^2s in the case of $\delta_{12} = 0$

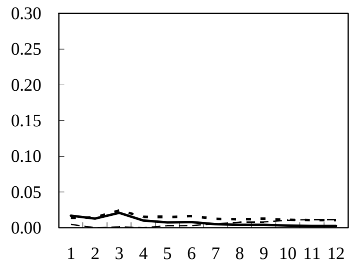
(i) Output growth rate



(ii) Inflation rate



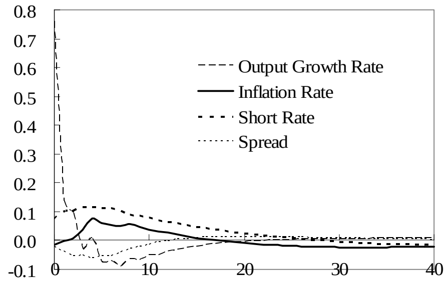
(iii) Short rate



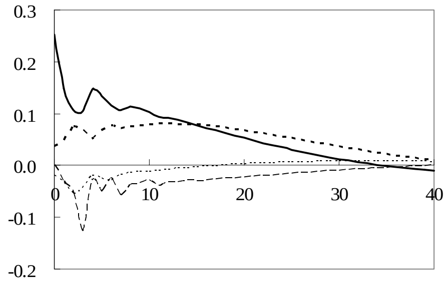
The sample and model-implied R^2s of OLS regressions (1)-(3) are reported. Left, center, and right panels correspond to output growth, inflation, and short rate regressions respectively. The horizontal axes correspond to forecasting horizons (quarters).

Figure 2: The impulse response functions of VAR variables

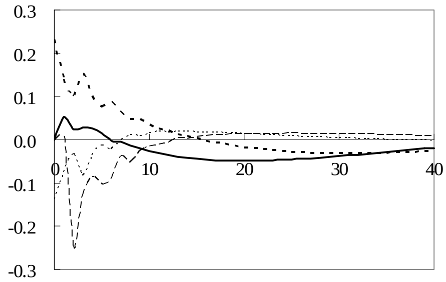
(2-a) Output growth shock



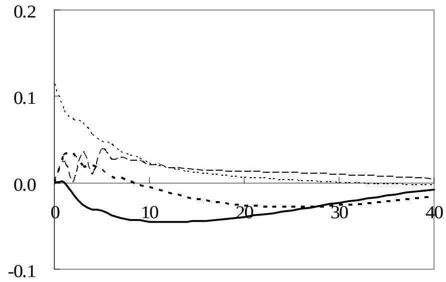
(2-b) Inflation shock



(2-c) Monetary policy shock



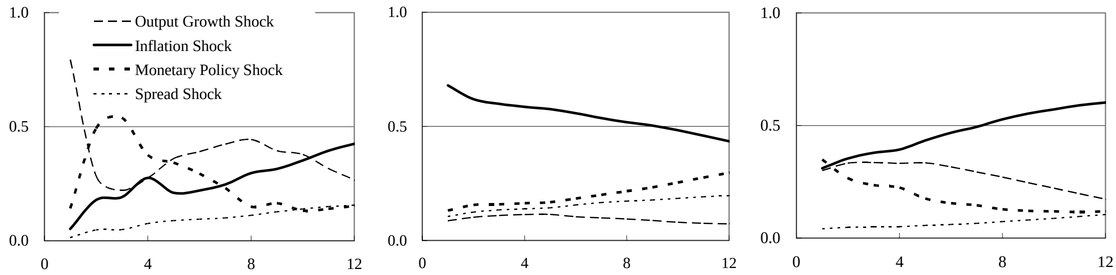
(2-d) Spread shock



The impulse response functions of VAR variables (percent per quarter) to one standard error exogenous shocks are reported. The horizontal axes correspond to horizons (quarters).

Figure 3: The variance decomposition of the optimal forecast of VAR variables

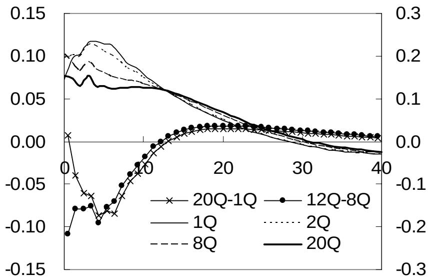
(3-a) Output growth rate (3-b) Inflation rate (3-c) Short rate



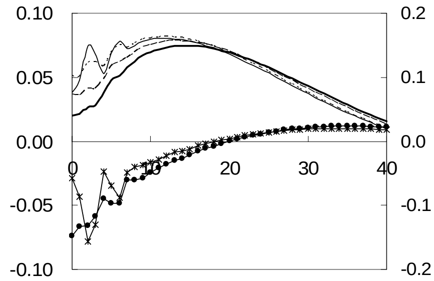
The variance decompositions of the optimal forecasts of VAR variables, in which the variances of the forecasts are normalized to unity, are reported. The horizontal axes correspond to forecasting horizons (quarters).

Figure 4: The impulse response functions of discount rates and the term spreads

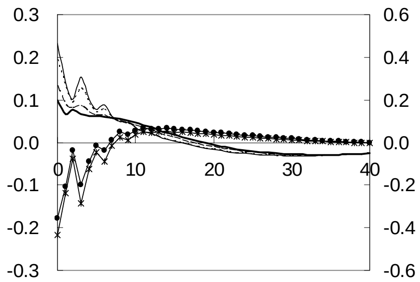
(4-a) Output growth shock



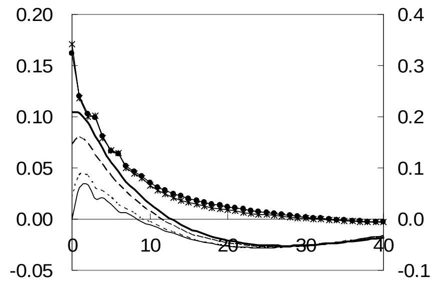
(4-b) Inflation shock



(4-c) Monetary Policy shock



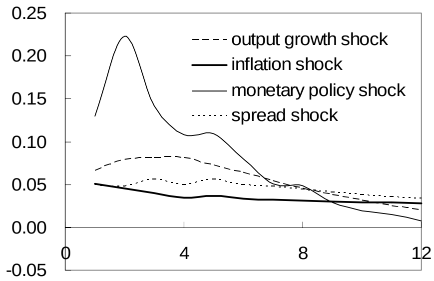
(4-d) Spread shock



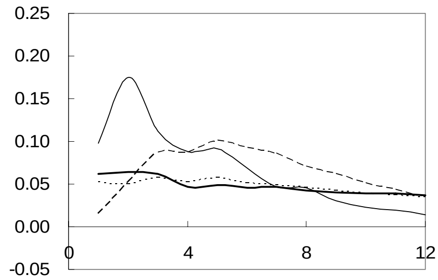
The impulse responses of the selected discount rates and term spreads to one standard error exogenous shocks are reported. Thin, dotted, broken, and thick lines correspond to 1Q, 2Q, 8Q, and 20Q rates respectively (left scale, percent per quarter). The impulse responses of the term spreads are normalized so that variances of the spreads equal unity (right scale). The horizontal axes correspond to horizons (quarters).

Figure 5: Decompositions of correlations between VAR variables and term spreads

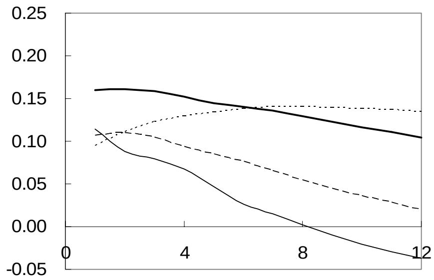
(5-a) Output growth rate and 20Q-1Q spread



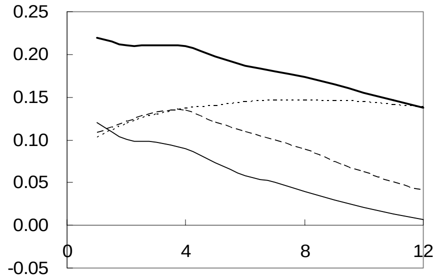
(5-d) Output growth rate and 12Q-8Q spread



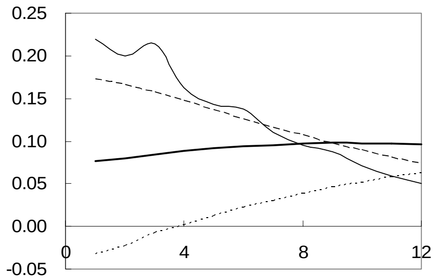
(5-b) Inflation rate and 20Q-1Q spread



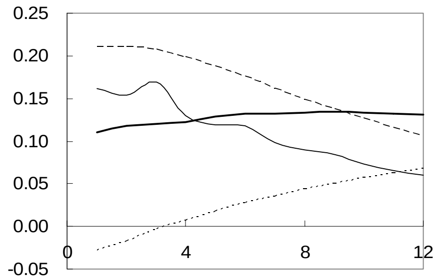
(5-e) Inflation rate and 12Q-8Q spread



(5-c) Short rate and 20Q-1Q spread



(5-f) Short rate and 12Q-8Q spread



The contributions of shocks to the correlations between future VAR variables and current term spreads are shown. Since the correlations of the term spreads with the inflation rate and the short rate are negative, Panels (5-b), (5-c), (5-e), and (5-f) are flipped. The horizontal axes correspond to forecasting horizons (quarters).